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LETTER TO THE EDITOR

FURTHER COMMENTS ON NONLINEAR VIBRATIONS OF SHELLS

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The recent letter by Dowell (1998) touched on several aspects of the literature and the physics of nonlinear vibrations of circular cylindrical shells, clarifying some of the issues while attempting to reconcile some hitherto contradictory findings, and at the same time outlining some of the unresolved issues and potentially profitable directions for further research.

The letter provided us with the impetus for some further thoughts on the subject, and here we offer some complementary discussion to a few of the points raised by Dowell (1998) and clarifications to our own work (Amabili *et al.* 1998).

The first issue is that of the magnitude of the nonlinear effects associated with forced motions of a simply supported, circular cylindrical shell. As pointed out by Dowell (1998), the degree of nonlinearity can be extremely weak: of the order of 0.5% (*vis-à-vis* the linear frequency) for amplitudes of the driven mode equal to the shell thickness. Such a small effect renders deliberation on whether the effect is hardening or softening almost immaterial.

Nevertheless, there exist other studies showing that this effect can be considerably stronger. For example, experiments by Chiba (1993a–c) show a maximum softening nonlinearity of 6% for empty shells and 8% for partially filled shells. Also, theoretical results by Gonçalves & Batista (1988) and Amabili *et al.* (1998) indicate softening nonlinearities of up to 5% for an empty and 20% for a completely water-filled shell for relatively modest amplitudes of the driven mode, equal to the shell thickness. Thus, the effect *can be* stronger

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and, in that light, warrants further consideration — making it, by the way, quite pertinent to wonder which parameters are responsible for this effect being found in one case to be so weak ($\sim 0{\cdot}5\%$) and in others more considerable (5%–20%).

The second point which we feel could benefit from further discussion is related to the modal expansion used in the analysis for simply supported shells. We wholly agree with Dowell that an expansion ideally involving all the linear modes could be the best one; we also believe that the mode with the same number of circumferential waves as the driven mode plus the axisymmetric modes are the most important. Furthermore, the modal expansion should satisfy (i) the geometric boundary conditions for the flexural displacement at the shell end, and (ii) continuity of the circumferential displacement. The authors' recent analysis (Amabili *et al.* 1998) appears to be the first to satisfy both of these conditions, whereas Evensen's (1967) appears to be deficient with regard to (i), and Dowell & Ventres' (1968) satisfies condition (ii) on the average only, for convenience.

Indeed, this is one of the strong aspects of our analytical approach; that and the introduction of an additional axisymmetric mode *vis-à-vis* previous attempts. Hence, we have proceeded to further improve it and have already conducted some work with a discretized system involving seven degrees of freedom. We have used this to investigate one of the problems also suggested by Dowell (1998): the nonlinear stability of simply supported cylindrical shells subjected to *axial fluid flow* (Amabili *et al.* 1999), the dynamics of which is both intricate and interesting.

References

Amabili, M., Pellicano, F. & Païdoussis, M. P. 1988 Nonlinear vibrations of simply supported, circular cylindrical shells, coupled to quiescent fluid. *Journal of Fluids and Structures* **12,** 883–918.

Amabili, M., Païdoussis, M. P. & Pellicano, F. 1999 Nonlinear dynamics and stability of circular cylindrical shells containing flowing fluid. *Journal of Sound and Vibration* (under review).

Chiba, M. 1993a Experimental studies on a nonlinear hydroelastic vibration of a clamped cylindrical tank partially filled with liquid. *ASME Journal of Pressure Vessel Technology* **115,** 381–388.

Chiba, M. 1993b Non-linear hydroelastic vibration of a cantilever cylindrical tank. I: Experiment (Empty case). *International Journal of Non-Linear Mechanics* **28,** 591–599.

Chiba, M. 1993c Non-linear hydroelastic vibration of a cantilever cylindrical tank II. Experiment (Liquid-filled case). *International Journal of Non-Linear Mechanics* **28,** 601–612.

Dowell, E. H. 1998 Comments on the nonlinear vibrations of cylindrical shells. *Journal of Fluids and Structures* **12,** 1087–1089.

Dowell, E. H. & Ventres, C. S. 1968 Modal equations for the nonlinear flexural vibrations of a cylindrical shell. *International Journal of Solids and Structures* **4,** 975–991.

Evensen, D. A. 1967 Nonlinear flexural vibrations of thin-walled circular cylinders. NASA TN D-4090.

Gonçalves, P. B. & Batista, R. C. 1988 Non-linear vibration analysis of fluid-filled cylindrical shells. *Journal of Sound and Vibration* **127,** 133–143.